**Investigation of the recovery efficiency in a halfbridge electromagnetic accelerator.**

Analytical solution describing recuperation of the energy for the diagonal halfbridge scheme is performed. Efficiency is analyzed for variety of conditions inherent to the real coilgun implementations.

This article includes:

- analytical relationship between the energies consumed from and returning to the capacitor of the halfbridge scheme;
- correlation between the physical and mathematical parameters of the halfbridge coilgun (geometry of an accelerating coil, projectile velocity and capacitor discharge level) enabling calculations for any specific system;
- analysis of the limitations of the power key maximum current and acceleration distance imposed on the range of parameters achievable in practical realizations of halfbridge coilguns;
- efficiency estimations made for two cases – "thick" and "thin" coils of a given geometry – in a velocity range actual for portative halfbridge coilguns.

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## 1. Introduction. Background information.

It is well known that during projectile is travelling inside the coil of an electromagnetic accelerator, there is a moment when the current flow through the coil must be stopped to prevent suckback. It can be performed by different ways, some of them are described here. The essence of all these methods is to dissipate the magnetic energy, accumulated in the coil, on any resistive element (e.g. the coil itself), that is permanently transferring it to heat. The feature of the halfbridge (or, more correctly, diagonal halfbridge) scheme is to pump this energy back to the capacitor by a specific reconnection of the coil. This allows us to use this energy in the next accelerating stage (if we assume a multistage system), or to reduce the capacitor recharging time (meaning one-stage coilgun). So, one may expect the increase of the total efficiency of multistage, and firing rate enhancement of single-stage accelerators.

When this article is being written, there are already some coilgun constructions using halfbridge topology (e.g. this and this), and they are indeed more effective than "traditional" ones. But there is still an obstacle in evaluating this effect quantitatively. Besides, as halfbridge is more complicated and expensive than simple (for example, with damping resistor-diode chain) realization, it is important to understand under what circumstances the application of the halfbridge is feasible, and when it is unreasonable.

The halfbridge scheme discussed is depicted in fig. 1. Blue arrows show the direction of current, and the ohmic resistance of the coil is also shown (it is considered to be the only resistive element in the circuit). On the left side the keys are open and current is increasing, on the right the keys are closed and current decaying to zero simultaneously flowing back into the cap. Hereinafter these two sequent conditions of the system are called "accumulation" and "recuperation", respectively. The keys are intentionally shown figuratively as SA1, SA2 switches – they can be MOSFETs or IGBTs; special methods may be used to commutate several stages by a single key; etc. Arbitrary dependences of current and capacitor voltage are presented under the figure 1.

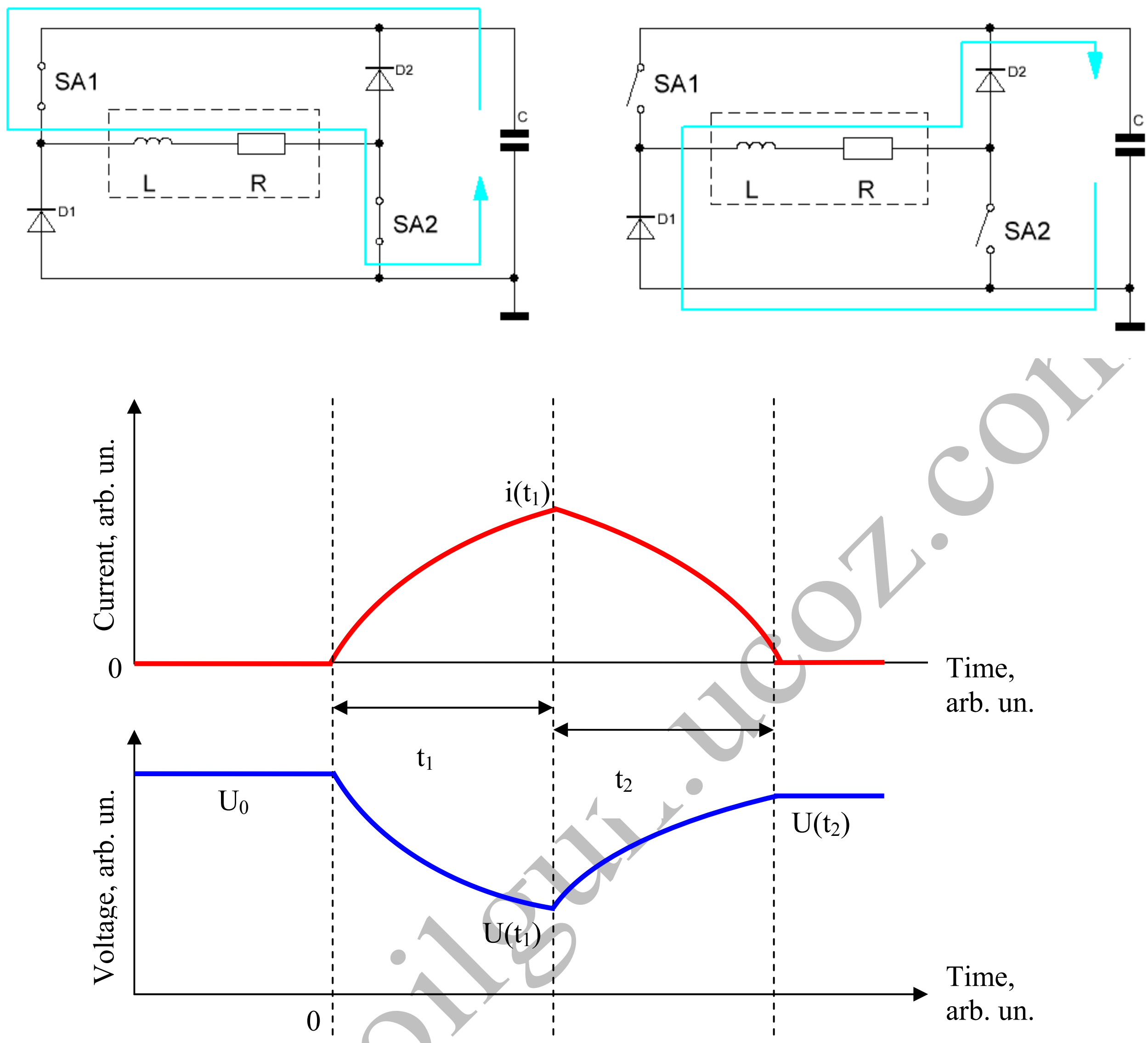


**Fig. 1.**



## 2. Basic equations.

To begin our analysis we should at first determine what we want to calculate. Recuperation efficiency is essential to be defined as a ratio of the energy returned to the cap to the end of recuperation cycle, to the energy consumed from the cap to the end of accumulation cycle:

$$\eta = \Delta Ec_2 / \Delta Ec_1 \qquad (1)$$

where $\Delta Ec_2 = ½\, C\, (U_c(t_2)^{\,2} - U_c(t_1)^{\,2})$ – capacitor energy gain in the 2nd cycle,
$\Delta Ec_1 = ½\, C\, (U_0^{\,2} - U_c(t_1)^{\,2})$ – capacitor energy loss in the 1st cycle.

(hereinafter we neglect the influence of the projectile on the energy distributions which is plausible for coilguns with their low efficiency)

Looking to the process in steps, we should at first note that the amount of energy available for "pushing" back into the cap is limited by the energy stored in magnetic field (current) of the coil on the moment when the keys are closed. I.e. on this moment already *the energy circulating in the circuit is only a part of one consumed from the cap*. It is obvious (although frequently forget about), because durng the first period $t_1$ a certain amount of heat is dissipated in ohmic resistance. This amount can be expressed in an arbitrary manner as

$$\eta_1 = \Delta E_L(t_1) / \Delta Ec_1 \qquad (2)$$

where the inductive energy is $\Delta E_L(t_1) = ½L \cdot i^2\ (t_1)$.

I should emphasize that determination of $\eta_1$ is very useful itself because this function *sets the limits for recuperation efficiency not only in halfbridge, but in any other circuit topology*. In other words, for any *RLC*-circuit we can at best save only a part of energy taken from capacitor, and this part is $\eta_1$. This coefficient can also be called "accumulation efficiency".

After the keys are closed, the current begins recirculation to the capacitor, simultaneously loosing a part of its energy on resistance, too. The efficiency of recuperation cycle can be expressed as

$$\eta_2 = \Delta Ec_2 / \Delta E_L(t_1) \qquad (3)$$

Thus, total efficiency according to (1) can be written as:

$$\eta = \eta_1 \cdot \eta_2 \qquad (4)$$

being the product of conversion efficiencies of capacitive energy into inductive one, and vise versa during the first and second stages of the process, respectively.

Now let's try to calculate $\eta_1$ and $\eta_2$ functions more definitely. To do this we are to use equations for current and voltage in *RLC*-circuit. They are given here. It is shown that *RLC*-circuit can be underdamped (when voltage and current oscillate from negative to positive values) or overdamped (having no oscillations), depending on the value of coefficient $k = 4L/R^2C$: if $k > 1$, the circuit is underdamped, if $k < 1$ – overdamped. Ii is also shown that all equations describing voltage and current time dependences in these two cases are equivalent with replacement of trigonometric functions to hyperbolic ones.

Suggesting the circuit to be underdamped (for certainty), we have for efficiency of accumulation cycle:

$$i_c(t) = \frac{2U_0}{R} \frac{e^{-\frac{t}{\tau_L}}}{\sqrt{k-1}} \sin\left(\frac{t}{\tau_L}\sqrt{k-1}\right) \qquad (5a)$$

$$U_c(t) = U_0 e^{-\frac{t}{\tau_L}} \left[ \cos\left( \frac{t}{\tau_L} \sqrt{k-1} \right) + \frac{1}{\sqrt{k-1}} \sin\left( \frac{t}{\tau_L} \sqrt{k-1} \right) \right] \quad (5b)$$

where $\tau_L = 2L/R$ – inductive constant of the circuit.

Substituting this equations to (2), we have following formula for the efficiency of the accumulation:

$$\eta_1 = \frac{k}{k-1} \frac{\exp\left( -2\frac{t_1}{\tau_L} \right) \sin^2\left( \sqrt{k-1}\frac{t_1}{\tau_L} \right)}{1 - \exp\left( -2\frac{t_1}{\tau_L} \right) \left[ \cos\left( \sqrt{k-1}\frac{t_1}{\tau_L} \right) + \frac{1}{\sqrt{k-1}} \sin\left( \sqrt{k-1}\frac{t_1}{\tau_L} \right) \right]^2} \quad (6)$$

Thus, we have solved the first part of the problem – we know what part of the energy wasted from capacitor is conserved in a magnetic field of the coil to the moment $t_1$. Now, the keys of the halfbridge are closed and current begins to flow back into the cap trying to restore its initial voltage. To determine an efficiency of this process, we can use this article again and find formulae describing current fall and voltage rise. Adapted to designations used here, they are:

$$i(t) = i(t_1) e^{-\frac{t}{\tau_L}} \left[ \cos(\frac{t}{\tau_L} \sqrt{k-1}) - \frac{1+m^{-1}}{\sqrt{k-1}} \sin\left( \frac{t}{\tau_L} \sqrt{k-1} \right) \right] \quad (7a)$$

$$U_c(t) = U_c(t_1) e^{-\frac{t}{\tau_L}} \left[ \cos\left( \frac{t}{\tau_L} \sqrt{k-1} \right) + \frac{1+km}{\sqrt{k-1}} \sin\left( \frac{t}{\tau_L} \sqrt{k-1} \right) \right] \quad (7b)$$

where

$$m = R \cdot i(t_1)/(2U_c(t_1)) \quad (7c)$$

and time $t$ is counted from the beginning of the recuperation cycle ($t_1$ according to our scale).

Coefficient $m$ determines the relevant coil current when keys are switched (for example, m <<1 in flyback converters, where energy is pumped to a capacitor by little portions). To calculate it in our case, one must substitute $i(t_1)$ and $U(t_1)$ values, determined by (5 a,b), to (7). This procedure gives

$$m(t_1) = \frac{\sin\left(\sqrt{k-1}\,\frac{t_1}{\tau_L}\right)}{\left[\left(\sqrt{k-1}\right)\cos\left(\sqrt{k-1}\,\frac{t_1}{\tau_L}\right) + \sin\left(\sqrt{k-1}\,\frac{t_1}{\tau_L}\right)\right]} \qquad (8)$$

Now lets write equation for energy increment in recuperation phase:

$$\Delta E_{c2} = ½C[U_c^2(t_2)\text{-}U_c^2(t_1)] = ½C\,U_c^2(t_1)[\,U_c^2(t_2)/\,U_c^2(t_1) - 1] \qquad (9)$$

Using eq. (3) and (9) and inductive energy $\Delta E_L(t_1) = ½L{\cdot}i^2\,(t_1)$ we can get for efficiency of the recuperation cycle:

$$\eta_2 = \frac{1}{k \cdot m^2}\left(\frac{U_c^2(t_2)}{U_c^2(t_1)} - 1\right) \qquad (10)$$

Here $U_c(t_2)$ is set by (7b), $U_c(t_1)$ – by (5b).

As we suggest an underdamped circuit, formulae (7 a,b) describe oscillation process. But there are diodes D1 and D2 in a halfbridge circuit (see fig. 1) which limit the oscillations after one half-wave, when current decays to zero and voltage reaches its maximum. So we should use an equation for duration of this half-wave (from here again):

$$t/\tau_L = \frac{1}{\sqrt{k-1}}\,arctg\left\{\frac{\sqrt{k-1}}{1+m^{-1}}\right\} \qquad (11)$$

Using this formula as an argument in (10) to find $U_c(t_2)$, we finally get for recuperation cycle efficiency

$$\eta_2 = \frac{1}{k \cdot m^2}\left(\exp\left(-\frac{2arctg\frac{\sqrt{k-1}}{1+m^{-1}}}{\sqrt{k-1}}\right)\left[\cos\left(arctg\frac{\sqrt{k-1}}{1+m^{-1}}\right) + \frac{1+mk}{\sqrt{k-1}}\cdot\sin\left(arctg\frac{\sqrt{k-1}}{1+m^{-1}}\right)\right]^2 - 1\right) \qquad (12)$$

Total recuperation efficiency (for 1 and 2 stages together), according to (4), is product of the functions (6) and (12). The equation got is rather complicated to be written here.

No we can get the most interesting – quantitative evaluations. It is not so easy as it seems at a first look, because the abstract mathematical values in formulae (4)-(12) are to be related to particular geometric and electric parameters of an electromagnetic launcher. The next chapter is dedicated to this procedure.

### 3. Correlation between the physical and mathematical parameters of the system.

To understand what result is given by the formulae elaborated, one should inspect them in a range of parameters inherent to the real coilgun constructions. But those parameters must be initially determined at first.

Lets look at equations (6), (8) and (12) which are a basis for calculations. They include two unknown values characterizing the system (damping coefficient $k$ and inductive constant $\tau_L$) and переменная $t_1$. How can this values be expressed by known parameters of a coilgun?

The easiest treatment is with the inductive constant – as it is shown in [1], $\tau_L$ is simply expressed through the coil length $l$, its outside and inside diameters $D$ and $d$, and winding density $a$ (i.e. ratio of wire diameters without and with isolation):

$$\tau_L = \frac{0{,}02a^2(D^2 - d^2)l}{\rho(13D + 18l - 7d)} \qquad (13)$$

Here $\tau_L$ is in microseconds, specific wire resistance $\rho$ in Ohm·cm (for example $\rho = 1.75 \cdot 10^{-6}$ Ohm·cm) and all geometric sizes – in cm.

Thus, the inductive constant is determined only by an accelerating coil geometry. To limit the range of the following calculations, I chose two cases – “thick” and “thin” coils with the next parameters:

Inside diameter $d$ = 8 mm,

For the thin coil: length $l$ = 24 mm, outside diameter $D$ = 16 mm;

For the thick coil: length $l$ = 16 mm, outside diameter $D$ = 24 mm (i.e. so called “ideal coil” is realized making a strongest magnetic field in its center with given power dissipation).

Let us set winding density $a$ = 0.85. This parameter defines an “empty” volume formed by a loose wire cores fit to each other.

Then for the “thick” copper coil we get $\tau_L$ = 1.24 ms, for the “thin” $\tau_L$ = 652 mcs (all calculated values are hereinafter rounded up to 3d significant digit).

Accumulation cycle duration $t_1$ can be assessed through the projectile velocity $v$. To do this we should make 3 assumptions:

1) Length of a projectile equals to the coil length (this case is known to provide highest efficiency);
2) Current begins to flow when the front end of the projectile goes into the coil;
3) Current decays to zero when the projectile is fully sucked into the coil.

Postulating this, we get

$t_1 + t_2 = l/v$ (14)

Taking into account that recuperation and accumulation cycle durations are approximately equal (which is proved by further calculations and shown in fig. 1), and velocity is approximately constant along the coil (which is almost always true for a multistage system), we have

$t_1 \approx ½\, l/v$ (15)

I have set a range of velocity from 10 to 150 m/s for the following calculations, which corresponds to $t_1$ = 53.3…….1200 mcs according to preset values of $l$.

The third parameter $k$ can be determined provided a new variable is included to the calculation $\Delta = U_c(t_1)/U_0$ characterizing a degree of capacitor discharge on the moment when accumulation is stopped.

According to common reflection the inclusion of $\Delta$ is reasonable, because the same amount of energy in the coil can be provided by two different ways – strongly discharging a little capacitor or slightly discharging a large one. It is clear that dynamics and effectiveness of the process in these two cases will differ as the voltage range applied to the coil varies.

For certainty let us take four values of discharge degree (it is convenient to be expressed in percents): 3%, 10%, 30% and 100% that corresponds to $\Delta$ values of 0.97, 0.9, 0.7 and 0. In the last case the capacitor is fully discharged, in the first one – very slightly discharged (as it is in multistage systems having a large capacitor and little energy waste on every stage).

Substituting $\Delta = U_c(t_1)/U_0$, $t_1$ and $\tau_L$ into (5b) we have a transcendent equation for $k$. Solving it (for example, graphically) we can get this last parameter. The values of $k$ for "thick" and "thin" coils produced by the procedure above are depicted in tables 1 and 2, respectively.

**Table 1. *k* for the "thick" coil ($\tau_L$ = 1,24 ms).**

| Velocity, m/s | $t_1$, mcs | *k* | | | |
|---|---|---|---|---|---|
| | | Δ = *0,97* | 0,9 | 0,7 | 0 |
| 10 | 800 | 0,215 | 0,727 | 2,29 | 9,70 |
| 20 | 400 | 0,714 | 2,41 | 7,56 | 30,7 |
| 30 | 267 | 1,50 | 5,07 | 15,8 | 63,4 |
| 50 | 160 | 3,96 | 13,4 | 41,7 | 165 |
| 75 | 107 | 8,61 | 29,1 | 90,6 | 357 |
| 100 | 80 | 15,2 | 51,3 | 160 | 627 |
| 125 | 64 | 23,5 | 79,4 | 247 | 970 |
| 150 | 53,3 | 33,8 | 114 | 354 | 1390 |

**Table 2. *k* for the "thin" coil ($\tau_L$ = 652 mcs).**

| Velocity, m/s | $t_1$, mcs | k | | | |
|---|---|---|---|---|---|
| | | $\Delta = 0{,}97$ | 0,9 | 0,7 | 0 |
| 10 | 1200 | 0,0449 | 0,153 | 0,491 | 2,50 |
| 20 | 600 | 0,121 | 0,410 | 1,30 | 5,73 |
| 30 | 400 | 0,233 | 0,786 | 2,49 | 10,4 |
| 50 | 240 | 0,561 | 1,90 | 5,95 | 24,3 |
| 75 | 160 | 1,18 | 3,96 | 12,4 | 49,7 |
| 100 | 120 | 2,02 | 6,78 | 21,2 | 84,3 |
| 125 | 96 | 3,06 | 10,4 | 32,3 | 128 |
| 150 | 80 | 4,34 | 14,7 | 45,8 | 181 |

Grey highlight of some cells will be explained further.

We see that *k* value corresponding to a fixed discharge grows with the velocity increases, and the growth is nonlinear: a decade speed growth produces two-decade *k* increase. Such a behavior can be explained rather easily. Lets assume we have a fixed-value capacitor. To discharge it to the same degree while velocity increases (i.e. discharge duration reduces) we should wind a coil with a thicker wire. Thus (as a coil geometry stays the same) the inductive constant $\tau_L$ will save its value, but the capacitive constant $\tau_C = RC/2$ will reduce proportionally to resistance, giving rise to *k*.

In a similar way *k* growth with $\Delta$ decrease at constant velocity is explained: to discharge the capacitance deeper and deeper during a fixed period, we should reduce its value which (provided all other parameters are constant) is equivalent to decrease of $\tau_C$ and increase of *k*.

So, we have associated the mathematical parameters *k*, $\tau_C$ and $t_1$ to the physical characteristics of a system – coil geometry, projectile velocity and discharge rate. Now we can calculate the recuperation efficiency, but let us before make another one mathematical digression

## 4. Influence of the physical limitations.

A legal doubt may arise: can we indeed get *k* values listed in tables 1 and 2 in a real coilgun construction?

The same question can be rephrased in a more general manner: all previous computations don't consider the physical restrictions of a coilgun construction. For example, power keys cannot sustain any current and voltage, a barrel cannot be infinite etc. Now, could the preset values of the parameters chosen for our calculations go beyond those limits?

To answer this question we should at first find out what we are limited by. First is maximum current and allowable voltage of the power keys. For available commercial semiconductors (at least on the moment when this article is being written) this values are limited by $I_{max}$ = 100 A and $U_{0max}$ = 1000 V.

Secondly, the acceleration path (which is approximately equal to the sum of the coils' longitudes in a multistage coilgun) isn't infinite as we assume a portable system. After some reflection it is obvious that it is equivalent to *restriction to a minimal energy* delivered to each stage of acceleration ($\Delta E_{C1}$ in our designations). Indeed, provided a total efficiency of a hypotetical accelerator is 10% (it is optimistic, compare to real efficiencies listed in Arsenal), we must spend 100 J from a cap to have 10 J in a projectile[1]. While we are able to place about 30 coils along 1m barrel (which looks maximal for a portable accelerator), we get $\Delta E_{C1} \approx 3$ J.

Changing initial conditions, one can get another value of $\Delta E_{C1}$. For instance, limiting the acceleration length by more realistic 30 cm, we have more rigid restriction $\Delta E_{C1} \approx 10$ J.

Are the new-introduced parameters enough to solve the problem stated in the beginning of this section? It appears that they are enough.

Let us remind equation for $k$:

$$k = \frac{\tau_L}{\tau_C} = \frac{2\tau_L}{R \cdot C} \tag{16}$$

As we fixed $\tau_L$ for the two cases earlier, minimal values $R_{min}$ and $C_{min}$ must be evaluated to determine maximum possible $k$ .

According to adopted designations, capacitor energy wasted during accumulation is $\Delta E_{C1} = \frac{CU_0^2}{2}[1-\Delta^2]$, so a minimal capacitance (for fixed energy waste) is

$$C_{\min} = \frac{2\Delta E_{C1}}{U_{0\max}^2[1-\Delta^2]} \tag{17}$$

On the other hand, a minimal ohmic resistance can be expressed via allowable current and voltage as

$$R = f(k, t_1, \tau_L) \cdot U_{0max}/I_{max} \tag{18}$$

where $f(k, t_1, \tau_L)$ is dimensionless function of the moment $t_1$ on $i_c(t_1)$ curve. As the current grows till the moment $t_{max}$ and then falls, one can distinguish two cases:

[1] It means approx. 60 m/s for a projectile suitable for the coils chosen in section 3.

1) $t_1 < t_{max}$. Here the maximal current is simply the current in $t_1$, and according to (5a):

$$f(k,t_1,\tau_L) = 2\frac{e^{-\frac{t_1}{\tau_L}}}{\sqrt{k-1}}\sin\left(\frac{t_1}{\tau_L}\sqrt{k-1}\right) \qquad (19a)$$

2) $t_1 \geq t_{max}$. Here the maximal current is fixed and equal to peak current reached in a *RLC*-circuit. Its value is given here in f. (7) and makes

$$f(k,t_1,\tau_L) = f(k) = 2\frac{e^{-\frac{arctg\sqrt{k-1}}{\sqrt{k-1}}}}{\sqrt{k-1}}\sin(arctg\sqrt{k-1}) \qquad (19b)$$

For clarity, the two occasions mentioned above are depicted in fig. 2.

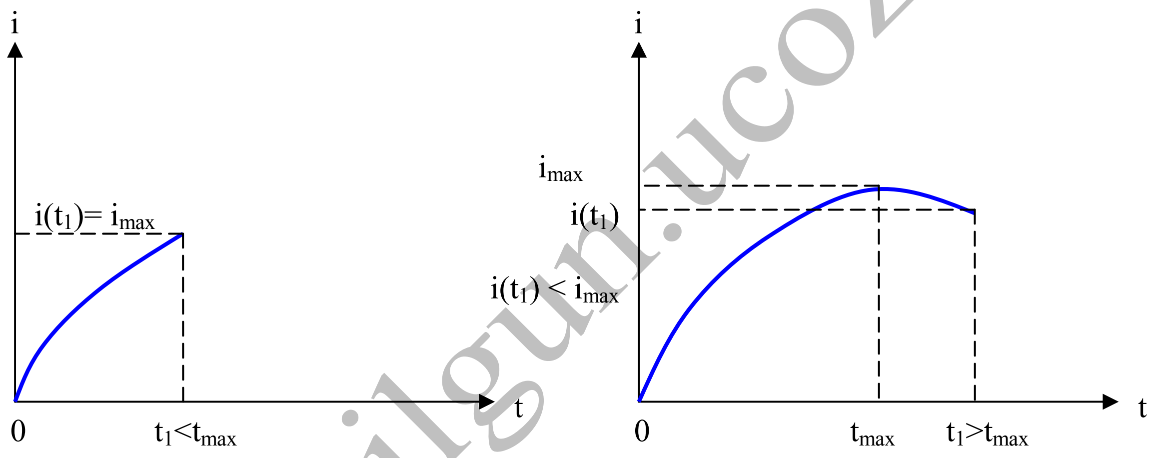


**Fig. 2. Possible cases for current switch-off. On the left – keys deactivated before current maximum (here an actual value of current is the maximal one), on the right – after the maximum (here an actual value of current is always less than the maximum).**

When making Table 1, I have conducted an additional calculation to define the moment according to $t_1$ on $i_c(t)$ curve. The cells with $t_1 \geq t_{max}$ (i.e. keys are deactivated after the current passed its maximum) are grey-colored.

Substituting (17) and (18) to (16) one can assess *maximum allowable k* for set conditions. In case it appears to be less than the value in appropriate cell of the tables 1 and 2, we must state, that we went out the limits of the coilgun's construction caused by the physical borders (barrel length and electrical parameters of the keys), and the further analysis is pointless, because we would assume a case known to be unreal.

Doing this, one will get tables 3 and 4. In fact they are tables 1 and 2 equipped with an additional color highlight. It is decoded in a following way: white background means that the case is real for the set conditions, yellow – unreal for barrel length less than 30 cm and maximum key current less than 100 A, red – unreal for barrel length less than 1 m, blue – unreal for barrel length less than 1 m and maximum key current less than 300 A.

**Table 3. Values of *k* with color highlight of its possible range for the "thick" coil.**

| Velocity, m/s | $t_1$, mcs | *k* | | | |
|---|---|---|---|---|---|
| | | $\Delta = 0{,}97$ | 0,9 | 0,7 | 0 |
| 10 | 800 | 0,215 | 0,727 | 2,29 | 9,70 |
| 20 | 400 | 0,714 | 2,41 | 7,56 | 30,7 |
| 30 | 267 | 1,50 | 5,07 | 15,8 | 63,4 |
| 50 | 160 | 3,96 | 13,4 | 41,7 | 165 |
| 75 | 107 | 8,61 | 29,1 | 90,6 | 357 |
| 100 | 80 | 15,2 | 51,3 | 160 | 627 |
| 125 | 64 | 23,5 | 79,4 | 247 | 970 |
| 150 | 53,3 | 33,8 | 114 | 354 | 1390 |

**Table 4. Values of *k* with color highlight of its possible range for the "thin" coil.**

| Скорость, м/с | $t_1$, мкс | k | | | |
|---|---|---|---|---|---|
| | | $\Delta = 0{,}97$ | 0,9 | 0,7 | 0 |
| 10 | 1200 | 0,0449 | 0,153 | 0,491 | 2,50 |
| 20 | 600 | 0,121 | 0,410 | 1,30 | 5,73 |
| 30 | 400 | 0,233 | 0,786 | 2,49 | 10,4 |
| 50 | 240 | 0,561 | 1,90 | 5,95 | 24,3 |
| 75 | 160 | 1,18 | 3,96 | 12,4 | 49,7 |
| 100 | 120 | 2,02 | 6,78 | 21,2 | 84,3 |
| 125 | 96 | 3,06 | 10,4 | 32,3 | 128 |
| 150 | 80 | 4,34 | 14,7 | 45,8 | 181 |



Initial voltage $U_0$ is 1000 V everywhere.

As we see, an interesting (although adverse for this article) result is got: for the chosen geometric parameters of the coils (see section 3) we must have more than 100A keys and a barrel longer than 1 m to reach ≈ 50 m/s velocity. To have 150 m/s the current up to 300 A is needed, which means especial modules or many power keys in parallel for every acceleration stage.

Giving a barrel of 30 cm and maximal key current 100 A the speed of 50 m/s is possible only for the "thin" coil and large capacitor (discharge depth no more than 3% per stage).

All calculations above are of course approximate, because we used rough values for the acceleration efficiency.

## 5. Calculation of the recuperation efficiency in different cases.

Now lets calculate the recuperation efficiency.

The efficiencies for accumulation and recuperation cycles $\eta_1$ and $\eta_2$ for the "thick" and "thin" coils in accordance to designations of the section 3 are given in tables 5-8. The total recuperation efficiency $\eta = \eta_1 \cdot \eta_2$ is depicted on fig. 3, dashed lines show the limits possible for the given parameters of the system (barrel length and key maximal current). According to the выводы of the previous section, these limits almost always don't depend on the discharge depth, i.e. they are vertical lines crossing the families of $\Delta$ curves at the same velocities. Exception is only the case of the "thin" coil for small discharge $\Delta = 0.97$.

**Table 5. Accumulation cycle efficiency for the "thick" coil.**

| Velocity, m/s | Δ = 0.97 | Δ = 0.9 | Δ = 0.7 | Δ = 0 |
|---|---|---|---|---|
| 10 | 0.461 | 0.453 | 0.427 | 0.274 |
| 20 | 0.663 | 0.658 | 0.639 | 0.523 |
| 30 | 0.756 | 0.752 | 0.738 | 0.649 |
| 50 | 0.844 | 0.841 | 0.831 | 0.772 |
| 75 | 0.892 | 0.89 | 0.883 | 0.841 |
| 100 | 0.918 | 0.916 | 0.911 | 0.879 |
| 125 | 0.934 | 0.932 | 0.928 | 0.902 |
| 150 | 0.944 | 0.943 | 0.94 | 0.917 |

**Table 6. Accumulation cycle efficiency for the "thin" coil.**

| Velocity, m/s | Δ = 0.97 | Δ = 0.9 | Δ = 0.7 | Δ = 0 |
|---|---|---|---|---|
| 10 | 0.172 | 0.165 | 0.142 | 0.025 |
| 20 | 0.351 | 0.342 | 0.315 | 0.159 |
| 30 | 0.477 | 0.469 | 0.443 | 0.293 |
| 50 | 0.628 | 0.622 | 0.602 | 0.479 |
| 75 | 0.728 | 0.723 | 0.707 | 0.61 |
| 100 | 0.787 | 0.783 | 0.77 | 0.692 |
| 125 | 0.824 | 0.821 | 0.811 | 0.745 |
| 150 | 0.851 | 0.848 | 0.839 | 0.783 |

**Table 7. Recuperation cycle efficiency for the "thick" coil.**

| Velocity, m/s | Δ = 0.97 | Δ = 0.9 | Δ = 0.7 | Δ = 0 |
|---|---|---|---|---|
| 10 | 0.682 | 0.675 | 0.654 | 0.427 |
| 20 | 0.763 | 0.758 | 0.741 | 0.598 |
| 30 | 0.813 | 0.809 | 0.795 | 0.692 |
| 50 | 0.869 | 0.866 | 0.857 | 0.791 |
| 75 | 0.905 | 0.903 | 0.896 | 0.851 |
| 100 | 0.925 | 0.924 | 0.919 | 0.884 |
| 125 | 0.939 | 0.937 | 0.933 | 0.905 |
| 150 | 0.948 | 0.947 | 0.943 | 0.92 |

**Table 8. Recuperation cycle efficiency for the "thin" coil.**

| Velocity, m/s | Δ = 0.97 | Δ = 0.9 | Δ = 0.7 | Δ = 0 |
|---|---|---|---|---|
| 10 | 0.618 | 0.613 | 0.597 | 0.236 |
| 20 | 0.65 | 0.644 | 0.622 | 0.351 |
| 30 | 0.687 | 0.681 | 0.659 | 0.442 |
| 50 | 0.747 | 0.741 | 0.723 | 0.567 |
| 75 | 0.797 | 0.792 | 0.777 | 0.662 |
| 100 | 0.831 | 0.828 | 0.815 | 0.726 |
| 125 | 0.856 | 0.853 | 0.842 | 0.769 |
| 150 | 0.874 | 0.871 | 0.862 | 0.8 |

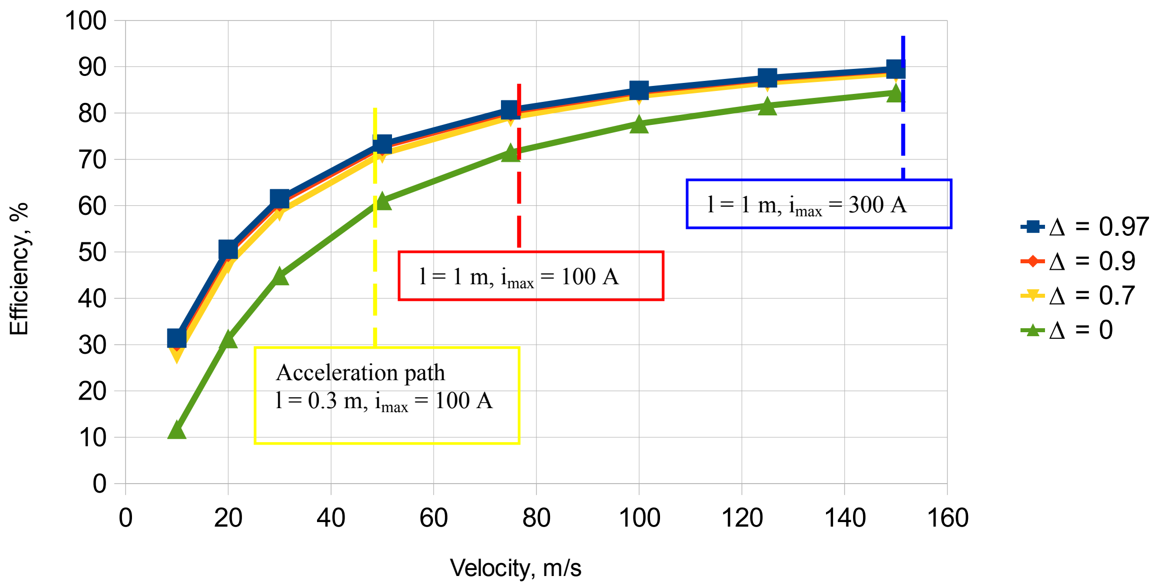


**Fig. 3(a). Total recuperation efficiency for the "thick" coil.**

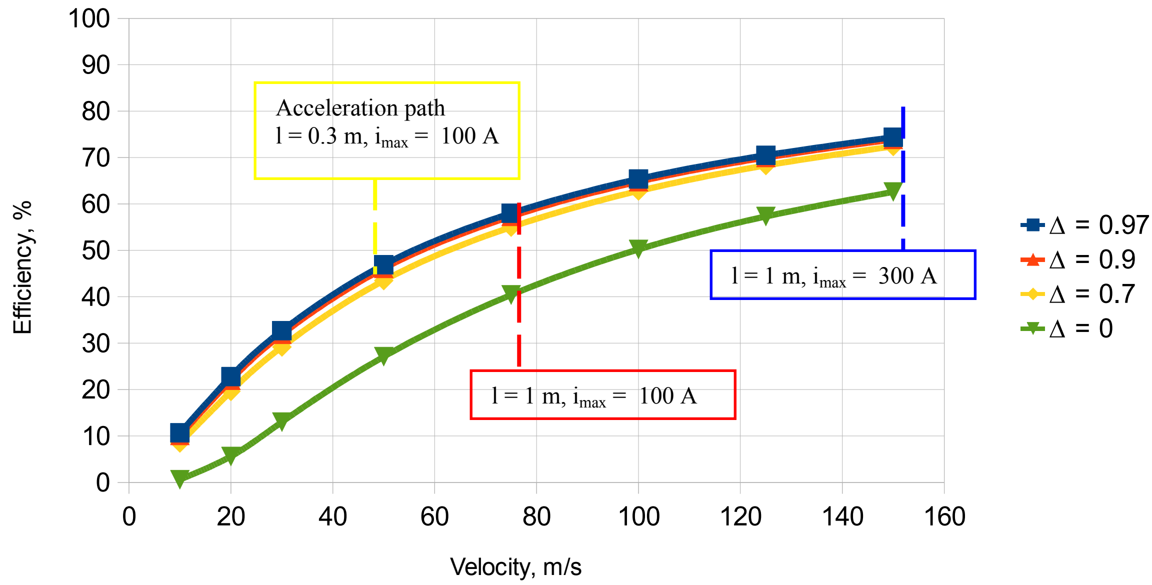


**Fig. 3(b). Total recuperation efficiency for the "thin" coil.**

CONCLUSIONS:

1) Total recuperation efficiency according to the conducted calculation appeared to be quite remarkable (tens percents) and increases significantly when the velocity of the projectile grows. $\eta$ reaches 80% in an actual for a multistage system speed range up to 100 m/s. *This shows that recuperation is in general mandatory when making high-efficiency coilgun.*

2) $\eta$ is always considerably larger for the "thick" coil than for the "thin" one. Only about 1/3 of the energy returns to a cap when the coil is "thin" and the velocity makes 30-40 m/s. *Usage of the halfbridge circuit is unreasonable in this case.*

One should mention that most home-made coilguns are working in just this velocity range (and they have coils close to our "thin" case). This explains their undistinguished characteristics (although higher than for the traditional constructions).

3) Discharge depth per stage in range of 3% to 30% doesn't affect on efficiency, but $\eta$ dramatically reduces when discharge becomes deeper. Within the mentioned range of $\Delta$ one can choose a value of capacitance guided by other limitations such as its ohmic resistance.

4) Efficiency of the recuperation cycle is always more than accumulation efficiency, but this difference becomes negligible for the short pulses (i.e. velocities more than $\approx$ 50 m/s).

Finally (especially for those who like to read such a long articles from the end ☺) I should note that all conclusions and conducted evaluations concern *the efficiency of the energy recuperation, not the acceleration efficiency* (we didn't take the projectile into our account at all). Approximate evaluation of the acceleration efficiency in a halfbridge scheme is given in the next section of the article, the exact result can be produced by modeling in FEMM-like programs. I hope to do this in later investigations.

## 6. Appendix.

6.1. Total acceleration efficiency in a halfbridge coilgun can be evaluated as follows. Let us suggest the acceleration efficiency in a traditional scheme (for example, with damping resistor) is $\eta_p$. Assuming the efficiency in a halfbridge circuit to be the same, we get the projectile energy increase when leaving the coil is $\eta_p \cdot \Delta E_{C1}$. Herewith the energy $\eta \cdot \Delta E_{C1}$ returns to a capacitor by the end of the recuperation cycle, i.e. total power waste from a capacitor is $(1-\eta)\cdot \Delta E_{C1}$. Dividing the projectile energy calculated above by this, we get the acceleration efficiency:

$$\eta_\Sigma = \frac{\eta_p}{1-\eta} \qquad (20)$$

For instance, if $\eta_p = 3$ %, we have for “thin” coil, small discharge and projectile velocity 30 m/s, $\eta_\Sigma \approx 4.3$ %, and for 100 m/s $\eta_\Sigma \approx 8.6$ % already. Actually the efficiency will be larger as $\eta_p$ grows with the velocity increase. This evaluation affirms the conclusion (2) of the previous section that the recuperative halfbridge circuit doesn’t allow a significant efficiency increase at low projectile speed.

6.2. Now let us check the suggestion about approximate equation of the accumulation and recuperation cycle durations, that was a basis for (15). To do it we should compare the duration of recuperation (11) to the $t_1$ value given in table 1. The result obtained is best to be given in a graphic way as $t_1/t_2$ dependence on accumulation cycle duration. It is done in fig. 4 for two values of damping coefficient 0.1 and 10 in a range of $t_1$ actual for suggested circumstances.

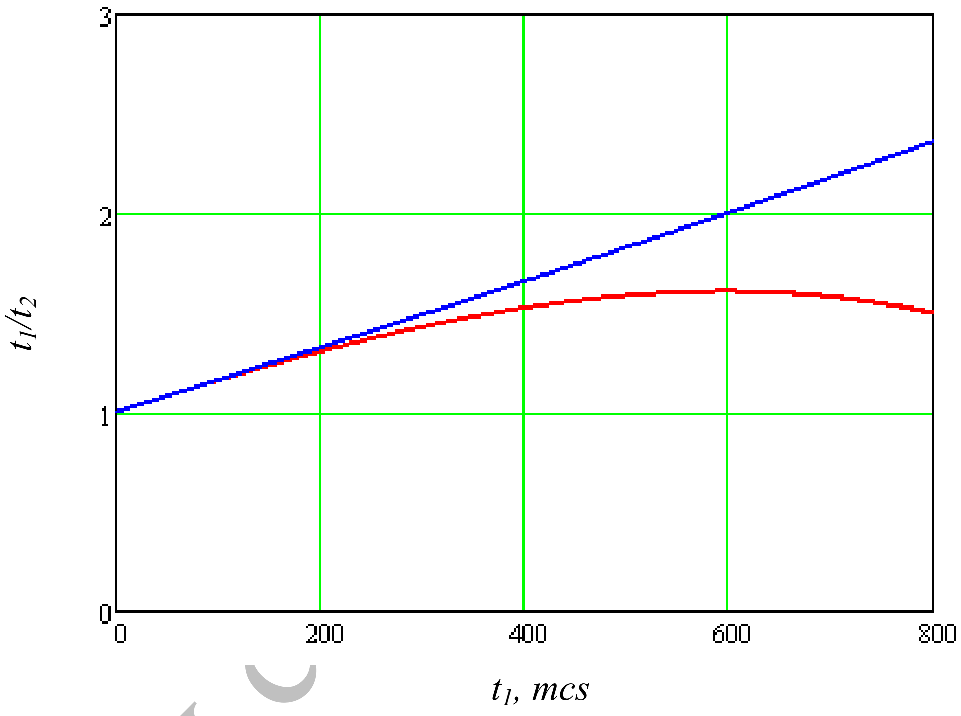


**Fig. 4. Accumulation to recuperation cycle duration ratio for $k$ = 0.1(—) and 10(—).**

Two conclusion may be made from this graph:

1) Accumulation lasts always some longer than recuperation.

2) Ratio of the durations is indeed close to one especially for the short cycles. It is closer to 1 for underdamped system ($k$ = 10), and a substantial (upto 2 and more) growth of $t_1/t_2$ value is only for an overdamped system and velocity range <20 m/s, which is not actual for a multistage construction.

Thus, eq. (15) can indeed be used for approximate evaluations.

6.3. The function (10) shows that the recuperation efficiency grows with a capacitor voltage. This is reasonable, because the duration of current decay $t_2$ reduces (as the ohmic power looses). But this leads to confession that the halfbridge circuit is not the best for recuperation – a scheme with coil discharging to a separate capacitor with high ( the higher – the better) voltage would be more optimal. One can even imagine a multistage coilgun with stages fed from caps of consequently growing voltage (fig. 5).

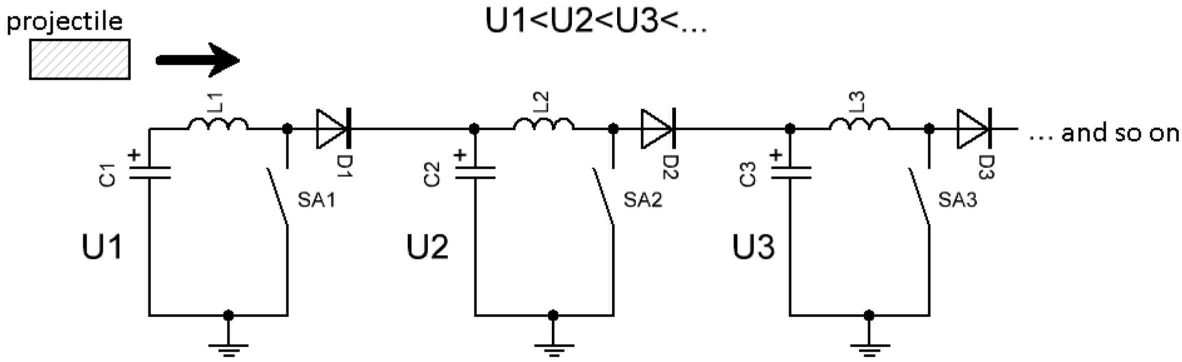


**Fig. 5. A hypothetical coilgun with increasing capacitor voltage on every stage.**

To provide a fast current decay (faster than in a conventional halfbridge) the voltage of the stage must be more than two times higher than the voltage of the previous one. One can suggest a following voltage sequence basing on a standard capacitor voltage row: 25 V, 63 V, 160 V, 400 V, 900 V (two 450 V caps in series).

This decision has an obvious disadvantage: the voltage on the last stages will be very high making connection and commutation difficult. As far as I know nobody has realized such a construction yet.

6.4. Recuperation efficiency increases when a peak current (or coefficient *m*, see eq. (7b)) falls. I.e. the less energy is consumed from a cap during the accumulation cycle, the more efficiently it is restored. However, this result stays in assumption that there are no other losses in a circuit but a coil's resistance. In a real construction the voltage drop exists on the keys, connection wires, diodes etc. Let us try to assess qualitatively an influence of one of these parasitic elements – diodes D1 and D2 (other losses are very circuit-specific and they are better to be evaluated in each case separately).

As a current flows through the diodes during recuperation only, their influence reduces to decrease of $\eta_2$. Assuming conventional silicon diodes one can evaluate a voltage drop to be about 0.7 V per single diode and 1.4 V in sum (or 0.6 V in sum for two Schottky diodes). This leads to a maximum voltage on a cap after recuperation to be no more than $U_0$-1.4V (or $U_0$-0.6V for Schottky diodes). To neglect these power losses one should discharge a cap to a value significantly more than 1.4V (0.6 V) on every acceleration stage. Taking one order margin, we have 14 V and 6 V, respectively. Thus we determined the *limitation on the minimal capacitor discharge per stage* (the limitation on the maximal one is set by the value of $\Delta$, see above ).

6.5. Functions (6) and (12) can be used to evaluate an efficiency of a flyback converter. Indeed, eq.(6) gives a direct evaluation for accumulation of energy in a choke (or in a primary winding of a transformer) $L_1$. To do this one must assume $C$ =

$\infty$, $L = L_1$ and $U_0$ to be power supply voltage of a converter (assuming that input shunting capacitor has an infinite value and inductance is constant).

Eq. (12) giving the recuperation cycle efficiency can be applied to a calculation of the flyback cycle, when the energy is discharged from a choke (primary winding) to a capacitor $C_{char}$. Now one should assume $C = C_{char}$, $U_c(t_1) = U(C_{char})$ and secondary winding inductance $L = L_2 = L_1 \cdot k^2$, where k is turns ratio in secondary and primary windings.

[1]. http://coilgun.ucoz.com/Mathematics_of_a_coilgun.pdf.